8-2 Special Right Triangles

What You'll Learn

- To use the properties of 45°-45°-90° triangles
- To use the properties of 30°-60°-90° triangles

. . . And Why

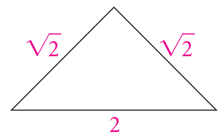
To find the distance from home plate to second base on a softball diamond, as in Example 3

✓ Check Skills You'll Need

GO for Help Lesson 1-6

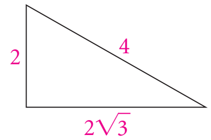
Use a protractor to find the measures of the angles of each triangle.

1.



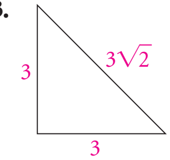
45, 45, 90

2.



30, 60, 90

3.

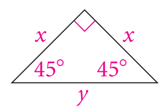


45, 45, 90

1 Using 45°-45°-90° Triangles

The acute angles of an isosceles right triangle are both 45° angles. Another name for an isosceles right triangle is a 45°-45°-90° triangle. If each leg has length x and the hypotenuse has length y, you can solve for y in terms of x.

$$x^2 + x^2 = y^2 \quad \text{Use the Pythagorean Theorem.}$$
$$2x^2 = y^2 \quad \text{Simplify.}$$
$$x\sqrt{2} = y \quad \text{Take the square root of each side.}$$



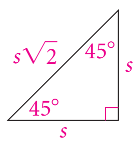
You have just proved the following theorem.

Key Concepts

Theorem 8-5 **45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

$$\text{hypotenuse} = \sqrt{2} \cdot \text{leg}$$



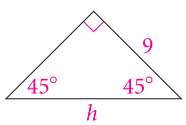
Test-Taking Tip

If you forget the formula for a 45°-45°-90° triangle, use the Pythagorean Theorem. The triangle is isosceles, so the legs have the same length.

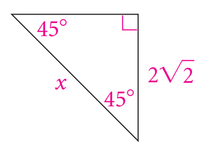
1 EXAMPLE Finding the Length of the Hypotenuse

Find the value of each variable.

a.



b.



$h = \sqrt{2} \cdot 9$ ⟵ hypotenuse $= \sqrt{2} \cdot$ leg ⟶ $x = \sqrt{2} \cdot 2\sqrt{2}$

$h = 9\sqrt{2}$ ⟵ Simplify. ⟶ $x = 4$

✓ Quick Check

1. Find the length of the hypotenuse of a 45°-45°-90° triangle with legs of length $5\sqrt{3}$. $5\sqrt{6}$

8-2

1. Plan

Objectives

1 To use the properties of 45°-45°-90° Triangles
2 To use the properties of 30°-60°-90° Triangles

Examples

1 Finding the Length of the Hypotenuse
2 Finding the Length of a Leg
3 Real-World Connection
4 Using the Length of One Side
5 Real-World Connection

Math Background

Professional Development

The ratio of the lengths of any two sides of a right triangle is a function of either acute angle. This can be proved using similarity theorems and is the basis for the six trigonometric functions. The fixed side-length ratios of 45°-45°-90° and 30°-60°-90° triangles, easily found by applying the Pythagorean Theorem, provide benchmark values for the trigonometric functions sine, cosine, and tangent of 30°, 45°, and 60° angles.

More Math Background: p. 414C

Lesson Planning and Resources

See p. 414E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

✓ Check Skills You'll Need

For intervention, direct students to:

Measuring Angles
Lesson 1-6: Example 2
Extra Skills, Word Problems, Proof Practice, Ch. 1

Differentiated Instruction Solutions for All Learners

Special Needs L1

For Example 3, have students check the answer by cutting out a 60-mm by 60-mm square. They fold it along its diagonal, and measure the length of the diagonal to the nearest millimeter.

learning style: tactile

Below Level L2

In the diagram for Theorem 8-6, construct a 30° angle adjacent to the 30° angle, using a leg as one side. Extend the base so that it intersects the new side. Discuss why this forms an equilateral triangle.

learning style: visual

2. Teach

Guided Instruction

1 EXAMPLE Technology Tip

Point out that using mental math is much faster than using a calculator for part b. The calculator answer also would be inexact, whereas squaring the square root of a number is always exact.

2 EXAMPLE Auditory Learners

Have several students explain aloud to the class how to rationalize a denominator.

PowerPoint

Additional Examples

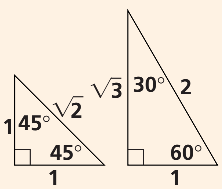
1 Find the length of the hypotenuse of a 45°-45°-90° triangle with legs of length $5\sqrt{6}$. $10\sqrt{3}$

2 Find the length of a leg of a 45°-45°-90° triangle with a hypotenuse of length 22. $11\sqrt{2}$

3 The distance from one corner to the opposite corner of a square playground is 96 ft. To the nearest foot, how long is each side of the playground? **68 ft**

Visual Learners

Suggest that students distinguish between the 45°-45°-90° and the 30°-60°-90° Triangle Theorems by using the "ratio" diagrams below.



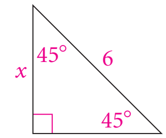
Error Prevention!

Whenever the length of a hypotenuse or longer leg of a 30°-60°-90° triangle is given, encourage students to find the length of the shorter leg first.

2 EXAMPLE Finding the Length of a Leg

Multiple Choice What is the value of x?

Ⓐ 3 Ⓑ $3\sqrt{2}$ Ⓒ 6 Ⓓ $6\sqrt{2}$



$6 = \sqrt{2} \cdot x$	**hypotenuse = $\sqrt{2}$ · leg**
$x = \frac{6}{\sqrt{2}}$	**Divide each side by $\sqrt{2}$.**
$x = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$	**Multiply by a form of 1.**
$x = 3\sqrt{2}$	**Simplify.**

The correct answer is B.

Quick Check 2 Find the length of a leg of a 45°-45°-90° triangle with a hypotenuse of length 10. $5\sqrt{2}$

When you apply the 45°-45°-90° Triangle Theorem to a real-life example, you can use a calculator to evaluate square roots.

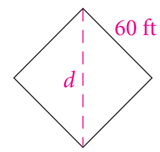
Real-World Connection

Careers Opportunities for coaching in women's sports have soared since the passage of Title IX in 1972.

3 EXAMPLE Real-World Connection

Softball A high school softball diamond is a square. The distance from base to base is 60 ft. To the nearest foot, how far does a catcher throw the ball from home plate to second base?

The distance d from home plate to second base is the length of the hypotenuse of a 45°-45°-90° triangle.



$d = 60\sqrt{2}$	**hypotenuse = $\sqrt{2}$ · leg**
$d = 84.852814$	**Use a calculator.**

On a high school softball diamond, the catcher throws the ball about 85 ft from home plate to second base.

Quick Check 3 A square garden has sides 100 ft long. You want to build a brick path along a diagonal of the square. How long will the path be? Round your answer to the nearest foot. **141 ft**

2 Using 30°-60°-90° Triangles

Another type of special right triangle is a 30°-60°-90° triangle.

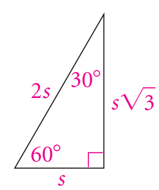
Key Concepts

Theorem 8-6 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

$$\text{hypotenuse} = 2 \cdot \text{shorter leg}$$

$$\text{longer leg} = \sqrt{3} \cdot \text{shorter leg}$$



Differentiated Instruction Solutions for All Learners

Advanced Learners L4
After students learn and apply Theorem 8-5, have them write a formula for the area of an isosceles right triangle whose hypotenuse has length *s*.

learning style: verbal

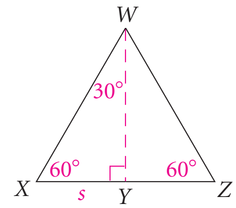
English Language Learners ELL
Ask students to complete each statement: *The shortest side of a triangle is always opposite the* smallest angle. In a 30°-60°-90° triangle, the shortest side is always opposite the 30° angle .

learning style: verbal

To prove Theorem 8-6, draw a 30°-60°-90° triangle using an equilateral triangle.

Proof

Proof of Theorem 8-6

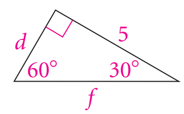


For 30°-60°-90° $\triangle WXY$ in equilateral $\triangle WXZ$, $\overline{WY}$ is the perpendicular bisector of $\overline{XZ}$.
Thus, $XY = \frac{1}{2}XZ = \frac{1}{2}XW$, or $XW = 2XY = 2s$.
Also,

$XY^2 + YW^2 = XW^2$	**Use the Pythagorean Theorem.**
$s^2 + YW^2 = (2s)^2$	**Substitute *s* for *XY* and 2*s* for *XW*.**
$YW^2 = 4s^2 - s^2$	**Subtract s^2 from each side.**
$YW^2 = 3s^2$	**Simplify.**
$YW = s\sqrt{3}$	**Find the square root of each side.**

The 30°-60°-90° Triangle Theorem, like the 45°-45°-90° Triangle Theorem, lets you find two sides of a triangle when you know the length of the third side.

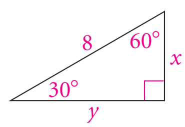
4 EXAMPLE Using the Length of One Side



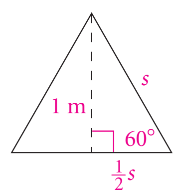
Algebra Find the value of each variable.

$5 = d\sqrt{3}$	**longer leg = $\sqrt{3}$ · shorter leg**
$d = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$	**Solve for *d*.**
$f = 2d$	**hypotenuse = 2 · shorter leg**
$f = 2 \cdot \frac{5\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$	**Substitute $\frac{5\sqrt{3}}{3}$ for *d*.**

Quick Check 4 Find the value of each variable. **$x = 4$, $y = 4\sqrt{3}$**



5 EXAMPLE Real-World Connection



Road Signs The moose warning sign at the left is an equilateral triangle. The height of the sign is 1 m. Find the length *s* of each side of the sign to the nearest tenth of a meter.

The triangle is equilateral, so the altitude divides the triangle into two 30°-60°-90° triangles as shown in the diagram. The altitude also bisects the base, so the shorter leg of each 30°-60°-90° triangle is $\frac{1}{2}s$.

$1 = \sqrt{3}\left(\frac{1}{2}s\right)$	**longer leg = $\sqrt{3}$ · shorter leg**
$\frac{2}{\sqrt{3}} = s$	**Solve for *s*.**
$s \approx 1.155$	**Simplify. Use a calculator.**

Each side of the sign is about 1.2 m long.

Quick Check 5 If the sides of the sign are 1 m long, what is the height? **about 0.9 m**

4 EXAMPLE Math Tip

Students can use the Pythagorean Theorem to check their work.

PowerPoint

Additional Examples

4 The longer leg of a 30°-60°-90° triangle has length 18. Find the lengths of the shorter leg and the hypotenuse. **shorter leg: $6\sqrt{3}$; hypotenuse: $12\sqrt{3}$**

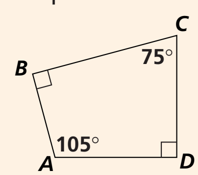
5 A garden shaped like a rhombus has a perimeter of 100 ft and a 60° angle. Find the perpendicular height between two bases. **21.7 ft**

Resources

- Daily Notetaking Guide 8-2 L3
- Daily Notetaking Guide 8-2—Adapted Instruction L1

Closure

In quadrilateral *ABCD*, $AD = DC$ and $AC = 20$. Find the area of *ABCD*. Leave your answer in simplest radical form.



$100 + 50\sqrt{3}$

3. Practice

Assignment Guide

1 A B 1-8, 21, 22, 26

2 A B 9-20, 23-25

C Challenge 27-28

Test Prep 29-32
Mixed Review 33-41

Homework Quick Check

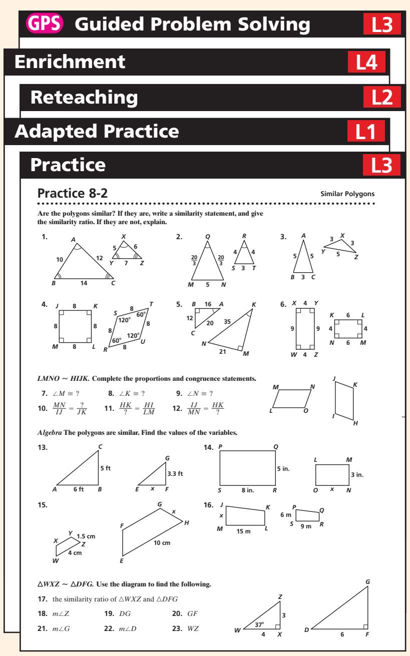
To check students' understanding of key skills and concepts, go over Exercises 4, 12, 18, 24, 25.

Exercise 7 Ask a volunteer to bring chopsticks to class and demonstrate how to use them.

Exercises 17–19 Students should first find any side length that can be derived using a given side. After the first length is found, the other lengths often fall into place.

Exercises 20–22 Each of these exercises requires constructing an altitude to form a rectangle.

Differentiated Instruction Resources



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

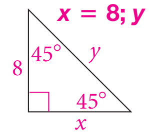
A Practice by Example

Example 1 (page 425)

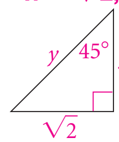
GO for Help

Find the value of each variable. If your answer is not an integer, leave it in simplest radical form.

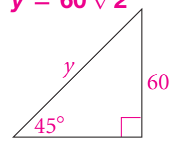
1. $x = 8;\ y = 8\sqrt{2}$



2. $x = \sqrt{2};\ y = 2$

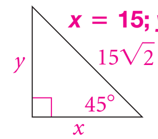


3. $y = 60\sqrt{2}$

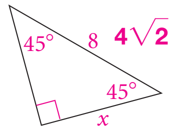


Examples 2 and 3 (page 426)

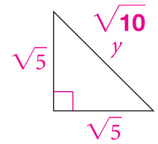
4. $x = 15;\ y = 15$

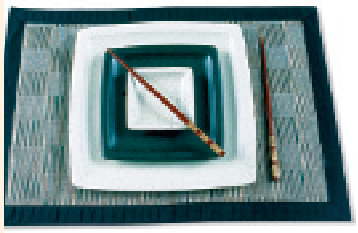


5. $4\sqrt{2}$



6. $\sqrt{10}$




Exercise 7

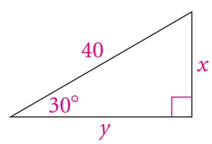
7. **Dinnerware Design** You are designing dinnerware. What is the length of a side of the smallest square plate on which a 20-cm chopstick can fit along a diagonal without any overhang? Round your answer to the nearest tenth of a centimeter. **14.1 cm**

8. **Helicopters** The four blades of a helicopter meet at right angles and are all the same length. The distance between the tips of two adjacent blades is 36 ft. How long is each blade? Round your answer to the nearest tenth. **25.5 ft**

Examples 4 and 5 (page 427)

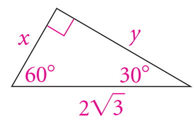
Algebra Find the value of each variable. If your answer is not an integer, leave it in simplest radical form.

9.



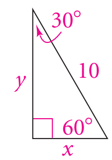
$x = 20;\ y = 20\sqrt{3}$

10.



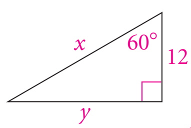
$x = \sqrt{3};\ y = 3$

11.



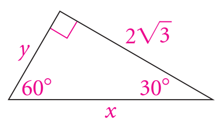
$x = 5;\ y = 5\sqrt{3}$

12.



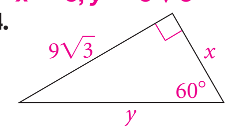
$x = 24;\ y = 12\sqrt{3}$

13.



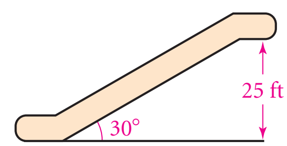
$x = 4;\ y = 2$

14.



$x = 9;\ y = 18$

15. **Architecture** An escalator lifts people to the second floor, 25 ft above the first floor. The escalator rises at a 30° angle. How far does a person travel from the bottom to the top of the escalator? **43 ft**



16. **City Planning** Jefferson Park sits on one square city block 300 ft on each side. Sidewalks join opposite corners. About how long is each diagonal sidewalk? **424 ft**

17. $a = 7;\ b = 14;\ c = 7;\ d = 7\sqrt{3}$

18. $a = 6;\ b = 6\sqrt{2};\ c = 2\sqrt{3};\ d = 6$

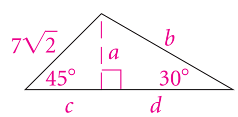
19. $a = 10\sqrt{3};\ b = 5\sqrt{3};\ c = 15;\ d = 5$

B Apply Your Skills

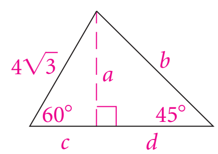
17–19. See above left.

Algebra Find the value of each variable. Leave your answer in simplest radical form.

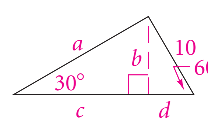
17.



18.



19.



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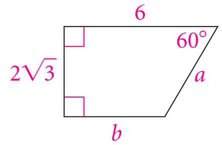
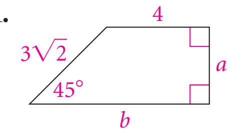
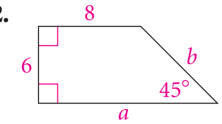
20. 

$a = 4$; $b = 4$

21. 

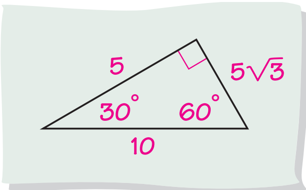
$a = 3$; $b = 7$

22. 

$a = 14$; $b = 6\sqrt{2}$

23. Rika; Sandra marked the shorter leg as opposite the 60° angle.

23. **Error Analysis** Sandra drew the triangle at the right. Rika said that the lengths couldn't be correct. With which student do you agree? Explain your answer.

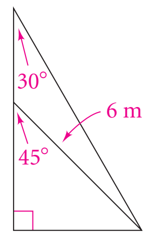


24. **Open-Ended** Write a real-life problem that you can solve using a 30°-60°-90° triangle with a 12 ft hypotenuse. Show your solution. **See margin.**


Exercise 25

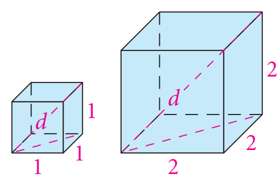
25. **Farming** A conveyor belt carries bales of hay from the ground to the barn loft 24 ft above the ground. The belt makes a 60° angle with the ground.
   a. How far does a bale of hay travel from one end of the conveyor belt to the other? Round your answer to the nearest foot. **28 ft**
   b. The conveyor belt moves at 100 ft/min. How long does it take for a bale of hay to go from the ground to the barn loft? **0.28 min**

26. **House Repair** After heavy winds damaged a farmhouse, workers placed a 6-m brace against its side at a 45° angle. Then, at the same spot on the ground, they placed a second, longer brace to make a 30° angle with the side of the house.
   a. How long is the longer brace? Round your answer to the nearest tenth of a meter. **8.5 m**
   b. How much higher on the house does the longer brace reach than the shorter brace? **3.1 m**



Challenge

27. **Geometry in 3 Dimensions** Find the length d, in simplest radical form, of the diagonal of a cube with sides of the given length. **See left.**
   a. 1 unit b. 2 units c. s units



27a. $\sqrt{3}$ units
b. $2\sqrt{3}$ units
c. $s\sqrt{3}$ units

28. **Constructions** Construct a 30°-60°-90° triangle given a segment that is
   a. the shorter leg.
   b. the hypotenuse.
   c. the longer leg. **See back of book.**

Test Prep

Multiple Choice

29. What is the length of a diagonal of a square with sides of length 4? **D**
   A. 2 B. $\sqrt{2}$ C. $2\sqrt{2}$ D. $4\sqrt{2}$

30. The longer leg of a 30°-60°-90° triangle is 6. What is the length of the hypotenuse? **H**
   A. $2\sqrt{3}$ B. $3\sqrt{2}$ C. $4\sqrt{3}$ D. 12

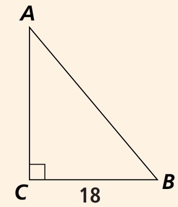
31. The hypotenuse of a 30°-60°-90° triangle is 30. What is the length of one of its legs? **D**
   A. $3\sqrt{10}$ B. $10\sqrt{3}$ C. $15\sqrt{2}$ D. 15

4. Assess & Reteach

Lesson Quiz

PowerPoint

Use $\triangle ABC$ for Exercises 1–3.



1. If $m\angle A = 45$, find AC and AB. **$AC = 18$; $AB = 18\sqrt{2}$**
2. If $m\angle A = 30$, find AC and AB. **$AC = 18\sqrt{3}$; $AB = 36$**
3. If $m\angle A = 60$, find AC and AB. **$AC = 6\sqrt{3}$; $AB = 12\sqrt{3}$**
4. Find the side length of a 45°-45°-90° triangle with a 4-cm hypotenuse. **$2\sqrt{2} \approx 2.8$ cm**
5. Two 12-mm sides of a triangle form a 120° angle. Find the length of the third side. **$12\sqrt{3} \approx 20.8$ mm**

Alternative Assessment

Have students use compass and straightedge to construct a large equilateral triangle with one altitude. Then have them explain how the three sides of one of the right triangles are related.

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 465
- Test-Taking Strategies, p. 460
- Test-Taking Strategies with Transparencies

24. Answers may vary. Sample: A ramp up to a door is 12 ft long. It has an incline of 30°. How high off the ground is the door? sol.: 6 ft

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 8-1 through 8-2.

Resources
Grab & Go
- Checkpoint Quiz 1

Short Response

[2] a. $4\sqrt{3}$ cm, OR about 6.9 in.
b. $8\sqrt{3}$ cm, OR about 13.9 in.
[1] one correct part

32. The vertex angle of an isosceles triangle is 120°. The length of its base is 24 cm.
a. Find the height of the triangle from the vertex angle to its base.
b. Find the length of each leg of the isosceles triangle.

Mixed Review

GO for Help

Lesson 8-1

For Exercises 33 and 34, leave your answers in simplest radical form.

33. A right traingle has a 6-in. hypotenuse and a 5-in leg. Find the length of the other leg. $\sqrt{11}$ in.

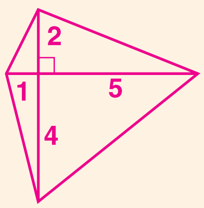
34. An isosceles triangle has 20-cm legs and a 16-cm base. Find the length of the altitude to the base. $4\sqrt{21}$ cm

Lesson 6-4

Determine whether each quadrilateral must be a parallelogram. If not, provide a counterexample.

35. The diagonals are congruent and perpendicular to each other. See margin.

35. no



36. Two opposite angles are right angles and two opposite sides are 5 cm long. yes

37. One pair of sides is congruent and the other pair of sides is parallel. no; an isosceles trapezoid

Lesson 4-3

Can you conclude that $\triangle TRY \cong \triangle ANG$ from the given conditions? If so, name the postulate or theorem that justifies your conclusion.

38. $\angle A \cong \angle T, \angle Y \cong \angle G, \overline{TR} \cong \overline{AN}$ 38. yes; AAS Thm.

39. $\angle T \cong \angle A, \angle R \cong \angle N, \angle Y \cong \angle G$ no

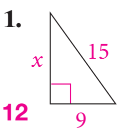
40. $\angle R \cong \angle N, \overline{TR} \cong \overline{AN}, \overline{TY} \cong \overline{AG}$ no

41. $\angle G \cong \angle Y, \angle N \cong \angle R, \overline{RY} \cong \overline{NG}$ yes; ASA Post.

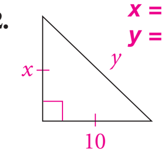
Checkpoint Quiz 1 — Lessons 8-1 through 8-2

x^2 **Algebra** **Find the value of each variable. Leave your answer in simplest radical form.**

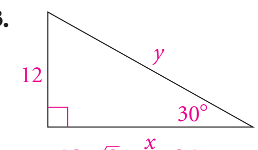
1. 12



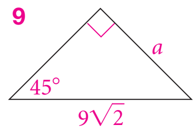
2. $x = 10$; $y = 10\sqrt{2}$



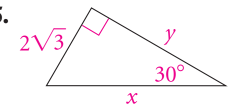
3. $x = 12\sqrt{3}$; $y = 24$



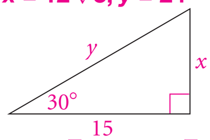
4. 9



5. $x = 4\sqrt{3}$; $y = 6$



6. $x = 5\sqrt{3}$; $y = 10\sqrt{3}$



The lengths of the sides of a triangle are given. Classify each triangle as *acute*, *obtuse*, or *right*.

7. 7, 8, 9 acute

8. 15, 36, 39 right

9. 10, 12, 16 obtuse

10. A square has a 40-cm diagonal. How long is each side of the square? Round your answer to the nearest tenth of a centimeter. 28.3 cm