8-6

1. Plan

Objectives
1 To describe vectors
2 To solve problems that involve vector addition

Examples
1 Describing a Vector
2 Describing a Vector Direction
3 Real-World Connection
4 Adding Vectors
5 Real-World Connection

Professional Development

Math Background

Scientific descriptions need to be precise and concise. Because vectors describe quantities with both magnitude and direction, they are especially useful in science. For example, the study of physics employs vectors extensively to describe force and velocity.

More Math Background: p. 414D

Lesson Planning and Resources

See p. 414E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:
Using the Pythagorean Theorem
Lesson 8-1: Example 2
Extra Skills, Word Problems, Proof Practice, Ch. 8

Vectors

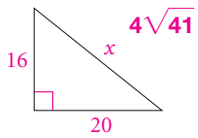
What You'll Learn
- To describe vectors
- To solve problems that involve vector addition

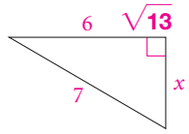
. . . And Why
To use vectors to describe the distance and direction of an airplane flight, as in Example 3

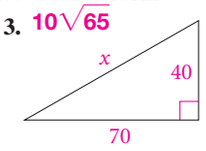
Check Skills You'll Need

GO for Help Lesson 8-1

Algebra Find the value of x. Leave your answers in simplest radical form.

1. 

2. 

3. 

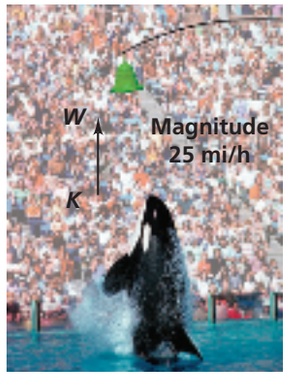
New Vocabulary
- vector
- magnitude
- initial point
- terminal point
- resultant

1 Describing Vectors

A **vector** is any quantity with magnitude (size) and direction. There are many models for a vector.

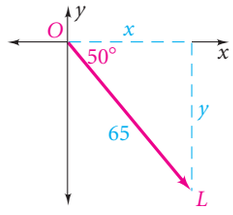
You can use an arrow for a vector as shown by the velocity vector $\overrightarrow{KW}$ in the photo. The **magnitude** corresponds to the distance from **initial point** K to the **terminal point** W. The direction corresponds to the direction in which the arrow points.

You can also use an ordered pair $\langle x, y \rangle$ in the coordinate plane for a vector. The magnitude and direction of the vector correspond to the distance and direction of $\langle x, y \rangle$ from the origin.



1 EXAMPLE Describing a Vector

Coordinate Geometry Describe $\overrightarrow{OL}$ as an ordered pair. Give the coordinates to the nearest tenth.



Use the sine and cosine ratios to find the values of x and y.

$\cos 50° = \frac{x}{65}$ $\qquad$ $\sin 50° = \frac{y}{65}$ $\qquad$ Use sine and cosine.

$x = 65(\cos 50°)$ $\qquad$ $y = 65(\sin 50°)$ $\qquad$ Solve for the variable.

≈ 41.78119463 $\qquad$ ≈ 49.7928888 $\qquad$ Use a calculator.

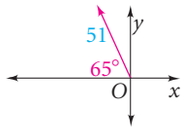
- L is in the fourth quadrant so the y-coordinate is negative. $\overrightarrow{OL} \approx \langle 41.8, -49.8 \rangle$.

Differentiated Instruction Solutions for All Learners

Special Needs L1
In Example 1, help students understand that the x-coordinate in $\langle 41.8, -49.8 \rangle$ is *positive* and the y-coordinate is *negative* because the direction from the origin to point L is *right* and *down*.
learning style: verbal

Below Level L2
Have students use centimeter graph paper to confirm the distance in Example 3 and use rulers and protractors to compare methods of describing the vector.
learning style: tactile

Quick Check 1 Describe the vector at the right as an ordered pair. Give the coordinates to the nearest tenth.
⟨−21.6, 46.2⟩



Real-World Connection

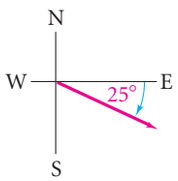
A velocity vector for a "bullet train" can have magnitude 275 km/h paired with any direction point on a compass.

In many applications of vectors, you use the compass directions north, south, east, and west to describe the direction of a vector.

2 EXAMPLE Describing a Vector Direction

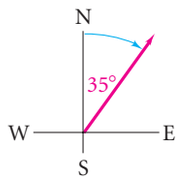
Use compass directions to describe the direction of each vector.

a.



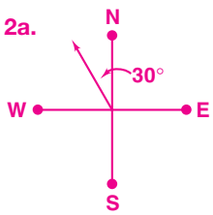
25° south of east

b.



35° east of north

2a.

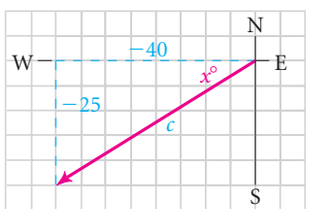


Quick Check 2 **a.** Sketch a vector that has the direction 30° west of north.
b. Critical Thinking Give a second description for the direction of this vector.
2b. 60° north of west

Example 3 shows how to describe a vector's magnitude and direction when you are given its description as an ordered pair.

3 EXAMPLE Real-World Connection

Aviation An airplane lands 40 km west and 25 km south from where it took off. The result of the trip can be described by the vector ⟨−40, −25⟩. Use distance (for magnitude) and direction to describe this vector a second way.



To find the distance, use the Distance Formula:

$d = \sqrt{(-40 - 0)^2 + (-25 - 0)^2}$

$d = \sqrt{1600 + 625}$ **Simplify.**

$d = \sqrt{2225}$

$d \approx 47.169906$ **Use a calculator to find the square root.**

To find the direction of the flight, find the angle of the vector south of west.

$\tan x^\circ = \frac{25}{40}$ **Find the tangent ratio.**

$x = \tan^{-1}\left(\frac{25}{40}\right)$ **Use the inverse of tangent.**

TAN^{-1} 25 ÷ 40 ENTER 32.005383 **Use a calculator.**

The airplane flew about 47 km at 32° south of west.

Quick Check 3 A small airplane lands at a point 246 mi east and 76 mi north of the point from which it took off. Describe the magnitude and the direction of its flight vector.
about 257 mi at 17° N of E

2. Teach

Guided Instruction

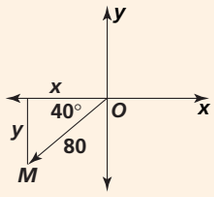
2 EXAMPLE Teaching Tip

In finding *35° east of north,* students should focus first on *due north* and then move 35° *east.* Encourage students to use a compass diagram when finding vector directions.

PowerPoint

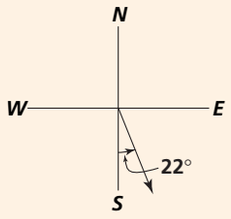
Additional Examples

1 Describe $\overrightarrow{OM}$ as an ordered pair. Give coordinates to the nearest tenth.



⟨−61.3, −51.4⟩

2 Use compass directions to describe the direction of the vector.



22° east of south

3 A boat sailed 12 mi east and 9 mi south. The trip can be described by the vector ⟨12, −9⟩. Use distance and direction to describe this vector a second way.
The boat sailed 15 mi at about 37° south of east.

Advanced Learners L4
After learning how to add vectors, students can investigate whether vector addition is commutative and associative.
learning style: verbal

English Language Learners ELL
Help students distinguish between *south of east* and *east of south*. In English, adjectives mostly precede the nouns they modify, such as *white house*. The first compass direction modifies the second primary direction.
learning style: verbal

Guided Instruction

Technology Tip
Have students check to see whether their calculators perform vector addition.

4 EXAMPLE **Connection to Physics**

Point out that vectors are used extensively in physics to find the resultant of several velocities, accelerations, or forces.

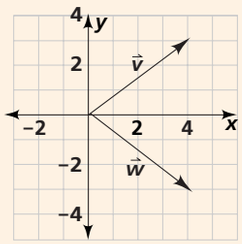
5 EXAMPLE **Alternative Method**

Point out that this example uses the Pythagorean Theorem, whereas Example 3 used the Distance Formula. Have students compare the two approaches.

PowerPoint

Additional Examples

4 Vectors $\vec{v}\ \langle 4, 3\rangle$ and $\vec{w}\ \langle 4, -3\rangle$ are shown below. Write $\vec{s}$, their sum, as an ordered pair.



$\vec{s}\ \langle 8, 0\rangle$

5 An airplane's speed is 250 mi/h in still air. The wind is blowing due east at 20 mi/h. If the airplane heads due north, what is its resultant speed and direction? Round answers to the nearest unit. **251 mi/h, 5° east of north**

Resources
- Daily Notetaking Guide 8-6 L3
- Daily Notetaking Guide 8-6—Adapted Instruction L1

Closure

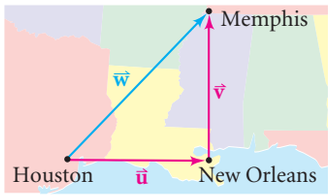
Sketch a vector with magnitude 50 and direction 30° west of north. Describe it as an ordered pair with coordinates rounded to the nearest tenth. **⟨−25, 43.3⟩; check that vectors are drawn from (0, 0) to (−25, 43.3).**

2 Adding Vectors

You can also use a single lowercase letter, such as $\vec{u}$, to name a vector.

This map shows vectors representing a flight from Houston to Memphis with a stopover in New Orleans. The vector from Houston to Memphis is called the sum, or **resultant,** of the other two vectors. You write this as

$$\vec{w} = \vec{u} + \vec{v}.$$

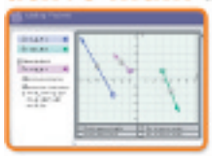


You can add vectors by adding their coordinates. You can also show the sum geometrically.

Key Concepts

Property	Adding Vectors
For $\vec{a} = \langle x_1, y_1\rangle$ and $\vec{c} = \langle x_2, y_2\rangle$, $\vec{a} + \vec{c} = \langle x_1 + x_2, y_1 + y_2\rangle$.	

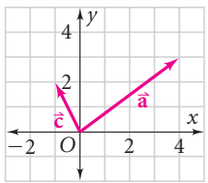
Online active math



For: Adding Vectors Activity
Use: Interactive Textbook, 8-6

4 EXAMPLE Adding Vectors

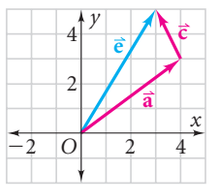
Vectors $\vec{a}\ \langle 4, 3\rangle$ and $\vec{c}\ \langle -1, 2\rangle$ are shown in the diagram. Write the sum of the two vectors as an ordered pair. Then draw $\vec{e}$, the sum of $\vec{a}$ and $\vec{c}$.



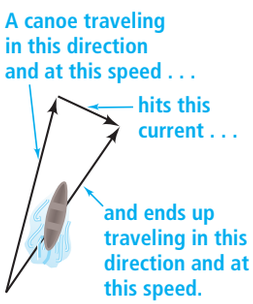
$$\vec{a} + \vec{c} = \langle 4, 3\rangle + \langle -1, 2\rangle$$
$$= \langle 4 + (-1), 3 + 2\rangle \quad \text{Add the coordinates.}$$
$$= \langle 3, 5\rangle \quad \text{Simplify.}$$

$\langle 3, 5\rangle$ is the resultant.

Draw $\vec{a}$ with its initial point at the origin. Then draw $\vec{c}$ with its initial point at the terminal point of $\vec{a}$. Finally, draw the resultant $\vec{e}$ from the initial point of $\vec{a}$ to the terminal point of $\vec{c}$.



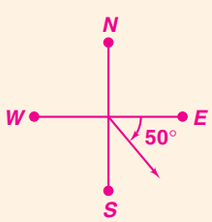
Quick Check 4 Write the sum of the two vectors $\langle 2, 3\rangle$ and $\langle -4, -2\rangle$ as an ordered pair. **⟨−2, 1⟩**

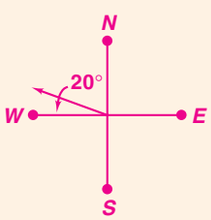


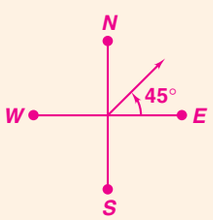
A vector sum can show the result of vectors that occur in sequence, such as in the airplane flight described above.

A vector sum can also show the result of vectors that act at the same time, such as when you row in a direction different from that of the current. See diagram at left.

The velocity of the canoe is the vector sum of the velocities of the paddlers and the stream.

7. 

8. 

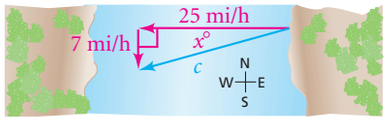
9. 

5 EXAMPLE **Real-World Connection**

Real-World Connection

Ferry service is essential in remote regions such as on the Mackenzie River in Canada's Northwest Territories.

Navigation A ferry shuttles people from one side of a river to the other. The speed of the ferry in still water is 25 mi/h. The river flows directly south at 7 mi/h. If the ferry heads directly west, what are the ferry's resultant speed and direction?



The diagram shows the sum of the two vectors. To find the ferry's resultant speed, use the Pythagorean Theorem.

$c^2 = 25^2 + 7^2$ **The lengths of the legs are 25 and 7.**

$c^2 = 674$ **Simplify.**

$c \approx 25.961510$ **Use a calculator.**

To find the ferry's resultant direction, use trigonometry.

$\tan x^\circ = \frac{7}{25}$ **Use the tangent ratio.**

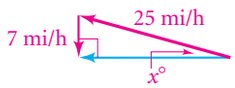
$x = \tan^{-1}\left(\frac{7}{25}\right)$ **Use the inverse of the tangent.**

$x \approx 15.642246$ **Use a calculator.**

The ferry's speed is about 26 mi/h. Its direction is about 16° south of west.

Quick Check

5 Critical Thinking Use the diagram to find the angle at which the ferry must head upriver in order to travel directly across the river.



about 16° north of west

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

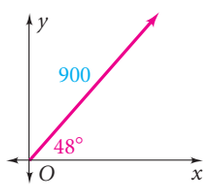
A Practice by Example

Example 1 (page 452)

Go for Help

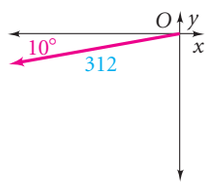
Describe each vector as an ordered pair. Give the coordinates to the nearest tenth.

1.



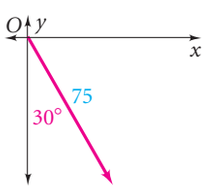
⟨602.2, 668.8⟩

2.



⟨−307.3, −54.2⟩

3.

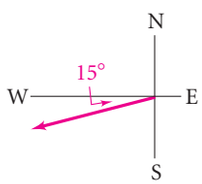


⟨37.5, −65.0⟩

Example 2 (page 453)

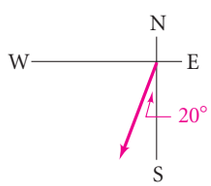
Use compass directions to describe the direction of each vector.

4.



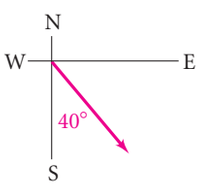
15° south of west

5.



20° west of south

6.



40° east of south

Sketch a vector that has the given direction. 7–12. See margin.

7. 50° south of east

8. 20° north of west

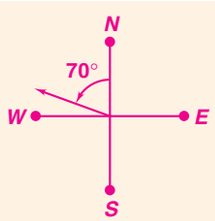
9. 45° northeast

10. 70° west of north

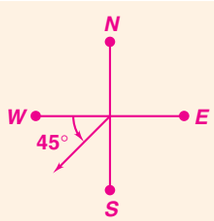
11. 45° southwest

12. 10° east of south

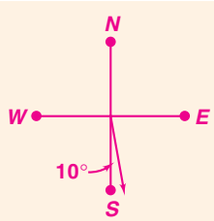
10.



11.



12.



3. Practice

Assignment Guide

1	A B	1-16, 29, 30, 32, 33, 40, 45, 46
2	A B	17-28, 31, 34-39, 41-44, 47, 48
C	Challenge	49-52
	Test Prep	53-55
	Mixed Review	56-60

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 2, 18, 31, 35, 40.

Error Prevention!

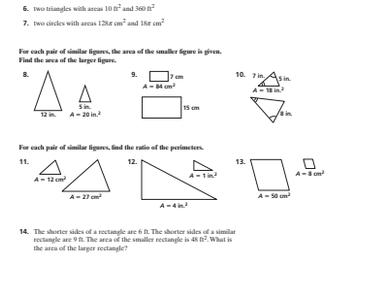
Exercises 2, 3 Students may forget to determine the signs of the coordinates. Remind students to check which quadrant contains the vector in the diagram.

Auditory Learners

Exercises 4–6 Have students work in small groups to name the direction of each vector using both the given angle and its complement.

Differentiated Instruction Resources

- GPS Guided Problem Solving L3
- Enrichment L4
- Reteaching L2
- Adapted Practice L1
- Practice L3


Practice 8-6 Perimeters and Areas of Similar Figures

Connection to History

Exercise 13 Point out that our modern Olympic Games, which started in 1896, originated in ancient Greece in 776 B.C.

Alternative Method

Exercise 14 Ask: *If* m *represents magnitude, what equation would you write to find* m *using the Distance Formula?* $m = \sqrt{300^2 + 640^2}$ *What equation would you write to find* m *using the Pythagorean Theorem?* $m^2 = 300^2 + 640^2$ Display the two equations, and have students explain why they are equivalent.

Tactile Learners

Exercises 17–22 Have students use pencils, straws, or other straight objects to model the vectors and their sums.

Diversity

Exercises 26, 27 Although there are many mathematics problems about boats and currents, many students are unfamiliar with the idea of forces pushing in different directions. Help students relate the problem to walking in a strong wind or swimming against a current.

Connection to Algebra

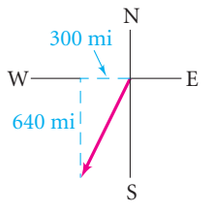
Exercise 31 Ask: *What algebraic property does the Parallelogram Rule establish?* **Commutative Property of Vector Addition**

Example 3 (page 453)

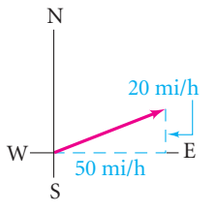
13. History Homing pigeons have the ability or instinct to find their way home when released hundreds of miles away from home. Homing pigeons carried news of Olympic victories to various cities in ancient Greece. Suppose one such pigeon took off from Athens and landed in Sparta, which is 73 mi west and 64 mi south of Athens. Find the distance and direction of its flight.
about 97 mi at 41° south of west

Find the magnitude and direction of each vector. 14–16. See left.

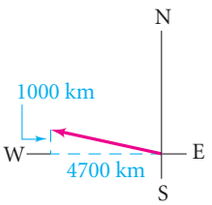
14.



15.



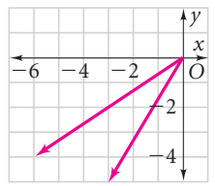
16.



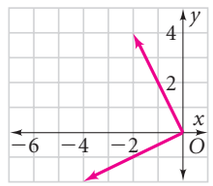
Example 4 (page 454)

In Exercises 17–22, (a) write the resultant as an ordered pair and (b) draw the resultant. 17–22. See margin.

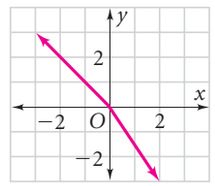
17.



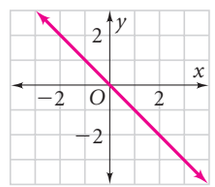
18.



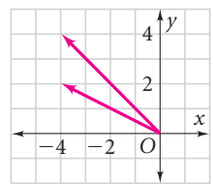
19.



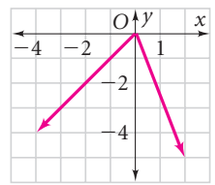
20.



21.



22.



Example 5 (page 455)

Write the sum of the two vectors as an ordered pair.

23. $\langle 2, 1\rangle$ and $\langle -3, 2\rangle$ **$\langle -1, 3\rangle$**

24. $\langle 0, 0\rangle$ and $\langle 4, -6\rangle$ **$\langle 4, -6\rangle$**

25. $\langle -1, 1\rangle$ and $\langle -1, 2\rangle$ **$\langle -2, 3\rangle$**

Navigation The speed of a powerboat in still water is 35 mi/h. It is traveling on a river that flows directly south at 8 mi/h.

26. The boat heads directly west across the river. What are the resulting speed and direction of the boat? Round answers to the nearest tenth. **See left.**

27. At what angle should the boat head upriver in order to travel directly west?

28. Aviation A twin-engine airplane has a speed of 300 mi/h in still air. Suppose this airplane heads directly south and encounters a 50 mi/h wind blowing due east. Find the resulting speed and direction of the plane. Round your answers to the nearest unit. **304 mi/h; 9° east of south**

B Apply Your Skills

29. Critical Thinking Valerie described the direction of a vector as 35° south of east. Pablo described it as 55° east of south. Could the two be describing the same vector? Explain. **See left.**

30. Error Analysis Ely says that the magnitude of vector $\langle 6, 1\rangle$ is 3 times that of vector $\langle 2, 1\rangle$ since 6 is 3 times 2. Explain why Ely's statement is incorrect.
$\langle 6, 1\rangle$ has mag. $\sqrt{37}$, but $\langle 2, 1\rangle$ has mag. $\sqrt{5}$.

14. about 707 mi; 65° south of west

15. about 54 mi/h; 22° north of east

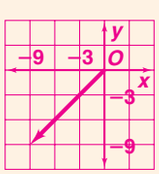
16. 4805 km; 12° north of west

26. 35.9 mi/h; 12.9° south of west

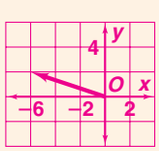
27. about 13.2° north of west

29. Yes; both vectors have the same direction, but could have diff. mag.

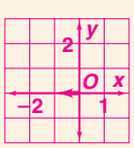
17. a. $\langle -9, -9\rangle$ b.



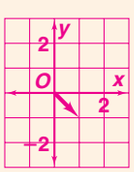
18. a. $\langle -6, 2\rangle$ b.



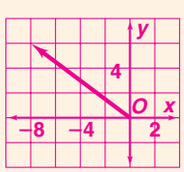
19. a. $\langle -1, 0\rangle$ b.



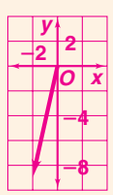
20. a. $\langle 1, -1\rangle$ b.

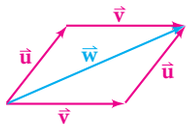


21. a. $\langle -8, 6\rangle$ b.

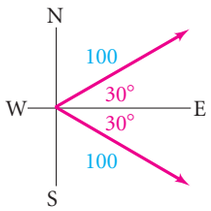


22. a. $\langle -2, -9\rangle$ b.





31. The diagram at the left shows that you can add vectors in any order. That is, $\vec{u} + \vec{v} = \vec{v} + \vec{u}$. Notice also that the four vectors shown in red form a parallelogram. The resultant $\vec{w}$ is the diagonal of the parallelogram. This representation of vector addition is called *The Parallelogram Rule*. **See margin.**



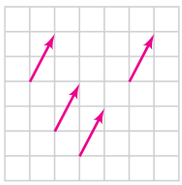
a. Copy the diagram at the right. Draw a parallelogram that has the given vectors as adjacent sides.

b. Find the magnitude and direction of the resultant. **about 173 due east**

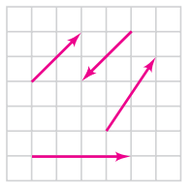
Problem Solving Hint

You can also model vector addition with the *Triangle Rule* as shown in Example 4, and by either triangular half of the diagram above.

32. Use the diagrams below to write a definition of *equal vectors*.

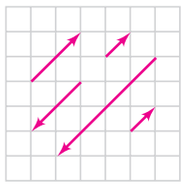


Equal vectors have the same mag. and direction.

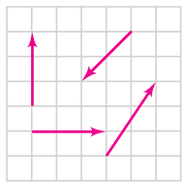


These vectors are equal. No two of these vectors are equal.

33. Use the diagrams below to write a definition of *parallel vectors*.



Vectors are ∥ if they have the same or opp. directions.



These vectors are parallel. No two of these vectors are parallel.

34. Multiple Choice A Red Cross helicopter takes off and flies 75 km at 20° south of west. There, it drops off some relief supplies. It then flies 130 km at 20° west of north to pick up three medics. What is the helicopter's distance from its point of origin? **C**

(A) 75 mi (B) 130 mi (C) 150 mi (D) 205 mi

35b. $\vec{a}$ and $\vec{c}$ have = mag. and opp. direction.

35. **a.** Find the sum of $\vec{a}$ and $\vec{c}$, where $\vec{a} = \langle 45, -60 \rangle$ and $\vec{c} = \langle -45, 60 \rangle$. **⟨0, 0⟩**

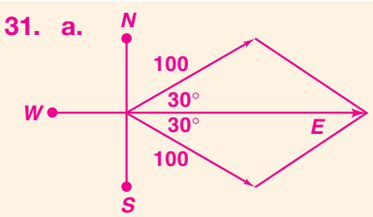
b. Writing Based on your answer to part (a), how can you describe $\vec{a}$ and $\vec{c}$?

36. Aviation In still air, the WP-3D (see below) flies at 374 mi/h. Suppose that a WP-3D flies due west and meets a hurricane wind blowing due south at 95 mi/h. What are the resultant speed and direction of the airplane to the nearest unit? **about 386 mi/h at 14° south of west**

Flying into a Hurricane

When most pilots hear a forecast for gale force winds, they don't think, "Time to fly." Then again, most pilots don't work for the National Oceanic and Atmospheric Administration. NOAA fly their four-engine WP-3D turboprops directly into hurricanes. These aircraft carry eight crew members, up to ten scientists, and a load of data-collection equipment. Some of this equipment is in the WP-3D's long "snout," which also serves as a lightning rod. In a routine flight, the WP-3D is struck by lightning three or four times. Surprisingly, small burn holes are the only damage from these strikes. To help overcome temporary blindness caused by lightning flashes, the pilot sets the cockpit lights at the brightest level.

31. a.



Math Tip

Exercise 33 After students write their definitions, point out that parallel vectors can have opposite directions.

Exercise 35 Point out that this exercise can be solved and analyzed without drawing a diagram.

Exercise 45 Ask: *Suppose* $\overrightarrow{AB}$ *describes walking due east at 3 mi/h. What does* $\overrightarrow{BA}$ *describe?* **walking due west at 3 mi/h** Have the class calculate the sum of $\overrightarrow{AB}$ and $\overrightarrow{BA}$. **0**

Visual Learners

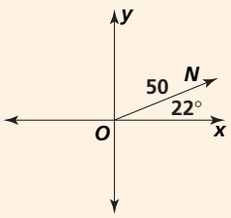
Exercise 49 Students may need help extending the Distance Formula to find the magnitude of a vector in three dimensions. If possible, provide a physical model to help explain the formula.

4. Assess & Reteach

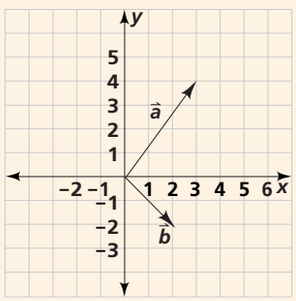
PowerPoint

Lesson Quiz

Use the diagram for Exercises 1 and 2.



1. Describe the vector as an ordered pair. Round coordinates to the nearest tenth. ⟨46.4, 18.7⟩
2. Use compass directions to describe the direction of $\overrightarrow{ON}$. 22° north of east
3. Iris rode her bike 30 mi south and 16 mi west of her home. Her trip can be described by the vector ⟨−16, −30⟩. Use distance and direction to describe the vector a second way. 34 mi at about 28° west of south
4. Write the vector $\vec{v} = \vec{a} + \vec{b}$ as an ordered pair.



⟨5, 2⟩

5. An airplane has a speed of 240 mi/h in still air. The plane heads due north and encounters a 30-mi/h wind blowing due east. Find the resultant speed and direction. Round to the nearest unit. 242 mi/h at 7° east of north

Alternative Assessment

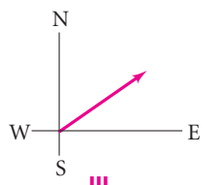
Have each student write a paragraph explaining how the vectors ⟨−6, 8⟩ and ⟨−8, 6⟩ are alike and how they are different.

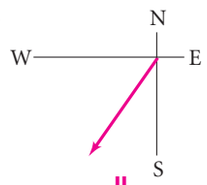
GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-0806

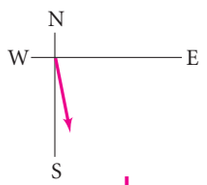
The vector ⟨−5, 5⟩ can be written as the *column matrix* $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$. Find the sum of the vectors in column matrix form.

37. $\begin{bmatrix} 2 \\ -4 \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ **38.** $\begin{bmatrix} 8 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $\begin{bmatrix} 11 \\ -5 \end{bmatrix}$ **39.** $\begin{bmatrix} 4 \\ -5 \end{bmatrix} + \begin{bmatrix} -5 \\ 5 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

40. Aviation An airplane takes off from a runway in the direction 10° east of south. When it reaches 5000 ft, it turns right 45°. It cruises at this altitude for 60 mi. Then it turns left 160°, descends, and lands. Match each vector with the appropriate portion of the flight.

I. 

II. 

III. 

A. The plane takes off. III B. The plane cruises. II C. The plane lands. I

41. Aviation The cruising speed of a Boeing 767 in still air is 530 mi/h. Suppose that a 767 is cruising directly east when it encounters an 80 mi/h wind blowing 40° south of west. a. See back of book. b. ⟨530, 0⟩; ⟨−61.3, −51.4⟩

a. Sketch the vectors for the velocities of the airplane and the wind.
b. Express both vectors from part (a) in ordered pair notation.
c. Find the sum of the vectors from part (b). ⟨468.7, −51.4⟩
d. Find the magnitude and direction of the vector from part (c). 471.5 mi/h at 6.3° south of east

Give the sum of $\vec{a}$ and $\vec{b}$. Show $\vec{a}$ and $\vec{b}$ and their sum in the coordinate plane.

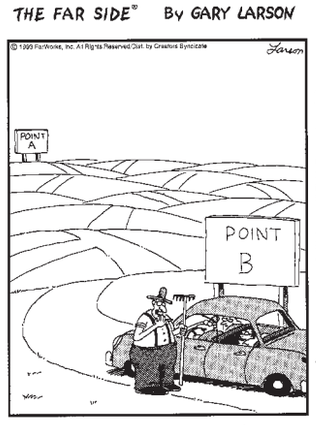
42. $\vec{a}$ ⟨−5, −2⟩, $\vec{b}$ ⟨2, −5⟩ **43.** $\vec{a}$ ⟨5, −2⟩, $\vec{b}$ ⟨−5, −2⟩ **44.** $\vec{a}$ ⟨5, −5⟩, $\vec{b}$ ⟨−2, 2⟩
42–44. See left.

45. Writing How are vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$ alike? How are they different? See left.

46. Open-Ended Name four other vectors with the same magnitude as ⟨−7, −24⟩. See left.

47. Navigation A fishing boat leaves its home port and travels 150 mi directly east. It then changes course and travels 40 mi due north. See left.

a. In what direction should the boat head to return to home port?
b. How long will the return trip take if the boat averages 23 mi/h?



"Well, lemme think. ... You've stumped me, son. Most folks only wanna know how to go the other way."

Exercise 45

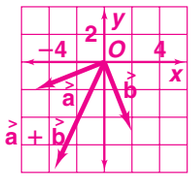
48. Navigation A boat left dock A, traveled north for 10 miles, then 45° east of north for 20 miles, and docked at B.

a. How far north did the boat travel? How far east did it travel? about 24.1 mi; about 14.1 mi
b. Find the magnitude and direction of the direct-path vector $\overrightarrow{AB}$. about 28 mi at about 30° east of north

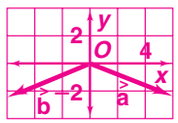
49. Geometry in 3 Dimensions A hot-air balloon traveled 2000 ft north and 900 ft east, while rising 400 ft. This trip can be described with the three-coordinate vector ⟨2000, 900, 400⟩. What is the magnitude of the vector? What is the angle of elevation of the balloon from its starting point? about 2229 ft; about 10°

Problem Solving Hint
In Exercise 40, remember that any vector is equal to one whose initial point is the origin.

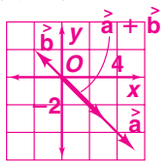
42. ⟨−3, −7⟩



43. ⟨0, −4⟩



44. ⟨3, −3⟩



45. The vectors have the same mag.; the vectors have opp. directions.

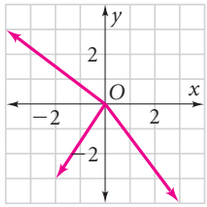
46. Answers may vary. Sample: ⟨7, 24⟩, ⟨−7, 24⟩, ⟨7, −24⟩, ⟨24, 7⟩

47a. about 15° south of west
b. about 6.7 h

C Challenge

50. a. Probability You choose two of the vectors at the right at random. Find the probability that the magnitude of their resultant vector is greater than that of the third vector. $\frac{2}{3}$

b. Open-Ended Draw three vectors of your own. Then do part (a) for your vectors. Check students' work.



51. Answers may vary. Sample: zero vector = ⟨0, 0⟩; it has mag. 0 and no direction.

51. Writing Think of the number zero and its properties. Define a *zero vector* and justify your definition. See left.

52. Aviation A helicopter starts at $(0, 0)$ and makes three parts of a flight represented by the vectors $\langle 10, 10\rangle$, $\langle 5, -4\rangle$, and $\langle -3, 5\rangle$, in that order.

a. If another helicopter starts at $(0, 0)$ and flies the same three parts in a different order, would it end in the same place? Justify your answer.

b. If yet another helicopter flew the three parts of the flight in a different order from the original trip, could the second part of the flight end at the same place as the second part of the original trip? Justify your answer.

a–b. See margin.

Test Prep

Multiple Choice

53. $\vec{c}$, $\vec{s}$, and $\vec{u}$ are vectors. $\vec{c} = \langle -8, 10\rangle$, $\vec{s} = \langle 0, -3\rangle$, and $\vec{u} = \vec{c} + \vec{s}$. What are the coordinates of $\vec{u}$? D

A. $\langle 7, -8\rangle$ **B.** $\langle -7, 8\rangle$ **C.** $\langle 8, -7\rangle$ **D.** $\langle -8, 7\rangle$

Short Response

54. A boat heads due south directly across a river at 30 ft/min. The river is flowing east at 20 ft/min.

a. What is the resultant speed of the boat? a–b. See margin.

b. What is the resultant direction of the boat?

Extended Response

55. A small aircraft is traveling east at 400 mi/h. It encounters a 50 mi/h wind blowing 30° west of south. a–d. See margin.

a. Sketch and label vectors for the velocities of the aircraft and the wind.

b. Express both vectors in ordered pair notation.

c. Find the sum of the vectors.

d. Find the magnitude and direction of the vector from part (c).

Mixed Review

GO for Help

Lesson 8-5

56. Indirect Measurement A hot-air balloon pilot sights the landing field from a height of 2000 ft. The angle of depression is 24°. To the nearest foot, what is the ground distance from the hot-air balloon to the landing field? 4492 ft

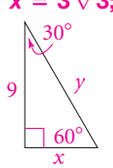
Lesson 8-2

Find the value of each variable.

57. x, $\sqrt{3}$, $\sqrt{6}$, 45°

58. $6\sqrt{2}$, 12, b, 45°, 45°

59. $x = 3\sqrt{3}$; $y = 6\sqrt{3}$



Lesson 6-1

60. Classify the quadrilateral with vertices $A(-1, -5)$, $B(6, -5)$, $C(9, 3)$, and $D(2, 3)$. parallelogram

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 465
- Test-Taking Strategies, p. 460
- Test-Taking Strategies with Transparencies

52. a. yes; when you add integers, which are the coordinates of the vectors, order is not important.

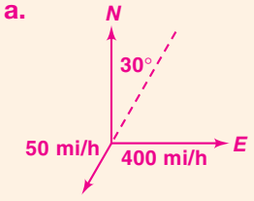
b. yes; if the first two vectors are the same, but in the opp. order

54. [2] a. about 36 ft/min

b. about 34° east of south

[1] correct speed OR correct direction

55. [4] a.



b. aircraft: ⟨400, 0⟩ wind: ⟨−25, −43.3⟩

c. ⟨375, −43.3⟩

d. about 377 mi/h, 6.6° south of east

[3] appropriate methods, but with one computational error

[2] correct speed of aircraft OR correct speed of wind

[1] correct speed of aircraft OR correct speed of wind without work shown