

Angles of Elevation and Depression

1. Plan

What You'll Learn

- To use angles of elevation and depression to solve problems

... And Why

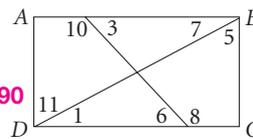
To use the angle of elevation to calculate the height of a natural wonder, as in Example 2

Check Skills You'll Need

Refer to rectangle $ABCD$ to complete the statements.

- $\angle 1 \cong \angle 7$
- $\angle 5 \cong \angle 11$
- $\angle 3 \cong \angle 6$
- $m\angle 1 + m\angle 5 = 90$
- $m\angle 10 + m\angle 3 = 180$
- $\angle 10 \cong \angle 8$

GO for Help Lesson 6-1



New Vocabulary • angle of elevation • angle of depression

1

Using Angles of Elevation and Depression

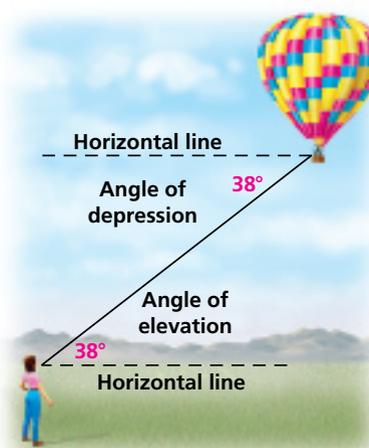
Suppose a person on the ground sees a hot-air balloon gondola at a 38° angle above a horizontal line.

This angle is the **angle of elevation**.

At the same time, a person in the hot-air balloon sees the person on the ground at a 38° angle below a horizontal line.

This angle is the **angle of depression**.

Examine the diagram. The angle of elevation is congruent to the angle of depression because they are alternate interior angles.



1 EXAMPLE Identifying Angles of Elevation and Depression

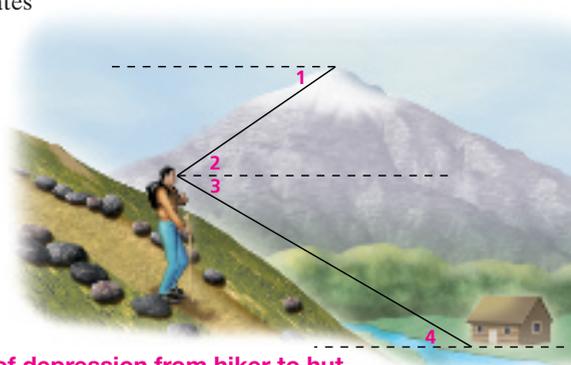
Describe each angle as it relates to the situation shown.

- $\angle 1$ $\angle 1$ is the angle of depression from the peak to the hiker.
- $\angle 4$ $\angle 4$ is the angle of elevation from the hut to the hiker.

Quick Check

- Describe each angle as it relates to the situation in Example 1.

- $\angle 2$
- $\angle 3$ \angle of depression from hiker to hut
 \angle of elevation from hiker to peak



Lesson 8-5 Angles of Elevation and Depression 445

Objectives

- To use angles of elevation and depression to solve problems

Examples

- Identifying Angles of Elevation and Depression
- Real-World Connection
- Real-World Connection



Math Background

Indirect measurement has been used since antiquity to measure distances that could not be measured directly. For example, Eratosthenes measured the Earth's circumference more than 2000 years ago, assuming the Earth to be round although subsequent scholars assumed it to be flat.

More Math Background: p. 414D

Lesson Planning and Resources

See p. 414E for a list of the resources that support this lesson.



Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Finding Measures of Angles

Lesson 3-1: Examples 4 and 5
Extra Skills, Word Problems, Proof Practice, Ch. 3

Applying the Triangle Angle-Sum Theorem

Lesson 3-4: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 3

Differentiated Instruction Solutions for All Learners

Special Needs L1

Use different colors to indicate angles of elevation and angles of depression. Then have students state the angle of elevation or depression *from* what object to what object.

learning style: visual

Below Level L2

Highlight the importance of parallel lines by having students copy the diagrams in Examples 1 and 3 and marking pairs of congruent angles in different colors.

learning style: visual

2. Teach

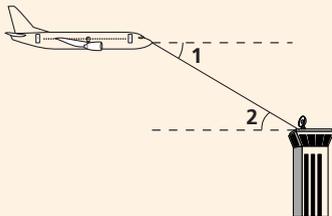
Guided Instruction

2 EXAMPLE Careers

Have students research the work description and tools of surveyors, including electronic distance measurement devices (EDMs).

PowerPoint Additional Examples

1 Describe $\angle 1$ and $\angle 2$ as they relate to the situation shown.



$\angle 1$ is the angle of depression;
 $\angle 2$ is the angle of elevation.

2 A surveyor stands 200 ft from a building to measure its height with a 5-ft tall theodolite. The angle of elevation to the top of the building is 35° . How tall is the building? **about 145 ft**

3 An airplane flying 3500 ft above ground begins a 2° descent to land at an airport. How many miles from the airport is the airplane when it starts its descent? **about 19 mi**

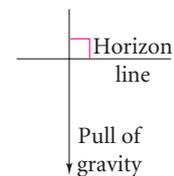
Resources

- Daily Notetaking Guide 8-5 **L3**
- Daily Notetaking Guide 8-5—Adapted Instruction **L1**

Closure

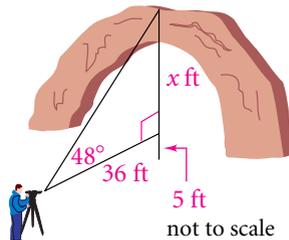
Two buildings are 30 ft apart. The angle of elevation from the top of one to the top of the other is 19° . What is their difference in height? **about 10 ft**

Surveyors use two instruments, the transit and the theodolite, to measure angles of elevation and depression. On both instruments, the surveyor sets the horizon line perpendicular to the direction of gravity. Using gravity to find the horizon line ensures accurate measures even on sloping surfaces.



2 EXAMPLE Real-World Connection

Surveying To find the height of Delicate Arch in Arches National Park in Utah, a surveyor levels a theodolite with the bottom of the arch. From there, she measures the angle of elevation to the top of the arch. She then measures the distance from where she stands to a point directly under the arch. Her results are shown in the diagram. What is the height of the arch?



$$\begin{aligned}\tan 48^\circ &= \frac{x}{36} \\ x &= 36(\tan 48^\circ) \\ 36 \text{ [TAN] } 48 \text{ [ENTER] } & 39.982051\end{aligned}$$

Use the tangent ratio.

Solve for x .

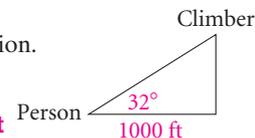
Use a calculator.

So $x \approx 40$. To find the height of the arch, add the height of the theodolite.

- Since $40 + 5 = 45$, Delicate Arch is about 45 feet high.

Quick Check

2 You sight a rock climber on a cliff at a 32° angle of elevation. The horizontal ground distance to the cliff is 1000 ft. Find the line-of-sight distance to the rock climber. **about 1179 ft**

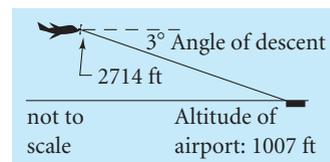


3 EXAMPLE Real-World Connection

Multiple Choice To approach runway 17 of the Ponca City Municipal Airport in Oklahoma, the pilot must begin a 3° descent starting from an altitude of 2714 ft. The airport altitude is 1007 ft. How many miles from the runway is the airplane at the start of this approach?

- (A) 3.6 mi (B) 5.7 mi (C) 6.2 mi (D) 9.8 mi

The airplane is $2714 - 1007$, or 1707 ft above the level of the airport.



$$\begin{aligned}\sin 3^\circ &= \frac{1707}{x} \\ x &= \frac{1707}{\sin 3^\circ} \\ 1707 \text{ [DIV] [SIN] } 3 \text{ [ENTER] } & 32616.2 \\ \text{[DIV] } 5280 \text{ [ENTER] } & 6.1773105\end{aligned}$$

Use the sine ratio.

Solve for x .

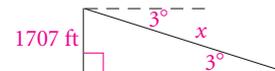
Use a calculator.

Divide by 5280 to convert feet to miles.

- The airplane is about 6.2 mi from the runway. The correct answer is C.

Quick Check

3 An airplane pilot sights a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's surface distance d from the raft? **about 6.2 km**



Advanced Learners **L4**
Challenge students to solve Example 3 using the cosine ratio.

English Language Learners **ELL**
Relate the meaning of *angle of depression* to depressions in the terrain or the Great Depression. Relate the meaning of *angle of elevation* to an elevator or elevation.

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 445)



Describe each angle as it relates to the situation in the diagram. 1–8. See margin.

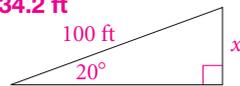
1. $\angle 1$ 2. $\angle 2$ 3. $\angle 3$ 4. $\angle 4$ 5. $\angle 5$ 6. $\angle 6$ 7. $\angle 7$ 8. $\angle 8$



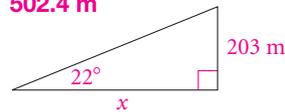
Example 2
(page 446)

Find the value of x . Round the lengths to the nearest tenth of a unit.

9. **34.2 ft**



10. **502.4 m**

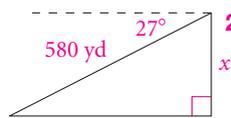


11. **Meteorology** A meteorologist measures the angle of elevation of a weather balloon as 41° . A radio signal from the balloon indicates that it is 1503 m from his location. To the nearest meter, how high above the ground is the balloon? **about 986 m**

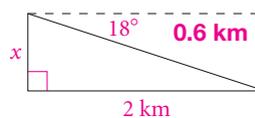
Example 3
(page 446)

Find the value of x . Round the lengths to the nearest tenth of a unit.

12. **263.3 yd**



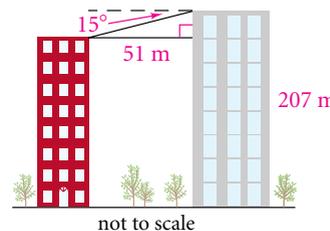
13. **0.6 km**



14. **Indirect Measurement** Miguel looks out from the crown of the Statue of Liberty approximately 250 ft above ground. He sights a ship coming into New York harbor and measures the angle of depression as 18° . Find the distance from the base of the statue to the ship to the nearest foot. **769 ft**

B Apply Your Skills

15. **Flagpole** The world's tallest unsupported flagpole is a 282-ft-tall steel pole in Surrey, British Columbia. The shortest shadow cast by the pole during the year is 137 ft long. To the nearest degree, what is the angle of elevation of the sun when the shortest shadow is cast? **64°**
16. **Engineering** The Americans with Disabilities Act states that wheelchair ramps can have a slope no greater than $\frac{1}{12}$. Find the angle of elevation of a ramp with this slope. Round your answer to the nearest tenth. **4.8°**
17. **Construction** Two office buildings are 51 m apart. The height of the taller building is 207 m. The angle of depression from the top of the taller building to the top of the shorter building is 15° . Find the height of the shorter building to the nearest meter. **about 194 m**



Visit: PHSchool.com
Web Code: aue-0805

Lesson 8-5 Angles of Elevation and Depression **447**

1. \angle of elevation from sub to boat
2. \angle of depression from boat to sub

3. \angle of elevation from boat to lighthouse
4. \angle of depression from lighthouse to boat
5. \angle of elevation from Jim to waterfall

6. \angle of elevation from Kelley to waterfall
7. \angle of depression from waterfall to Jim
8. \angle of depression from waterfall to Kelley

3. Practice

Assignment Guide

1 A B 1-28

C Challenge 29-30

Test Prep 31-34
Mixed Review 35-40

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 10, 14, 19, 24, 26.

Error Prevention!

Exercise 14 Some students may think the angle of depression is the angle between the vertical segment to the ground and the ship. Ask each student to draw a diagram that represents the situation in the exercise and then compare diagrams with a partner. Emphasize that one side of an angle of depression or of an angle of elevation must be horizontal.

Differentiated Instruction Resources

GPS Guided Problem Solving	L3
Enrichment	L4
Reteaching	L2
Adapted Practice	L1
Practice	L3

Practice 8-5 Proportions in Triangles

Use the figure at the right to complete each proportion.

- $\frac{AB}{AC} = \frac{AD}{AE}$
- $\frac{CE}{CD} = \frac{AE}{AD}$
- $\frac{AD}{AC} = \frac{AE}{AB}$
- $\frac{DE}{CE} = \frac{AD}{AB}$
- $\frac{AD}{AE} = \frac{CD}{CE}$
- $\frac{AD}{AC} = \frac{DE}{CE}$

Algebra Find the values of the variables.

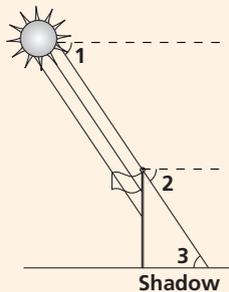
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Algebra Solve for x .

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Connection to Physics

Exercise 15 The sun's great distance from Earth explains why its rays are considered to be parallel. Copy the diagram below on the board to clarify how the angle of depression from the sun to the top of the flagpole relates to the angle of elevation from the end of the shadow to the top of the flagpole. Point out that as the position of the sun changes during the day, the angle of depression from the sun to the top of the flagpole changes. Discuss how the length of the shadow is longer when the sun is lower in the sky and shortest when the sun is highest in the sky.



Connection to Language Arts

Exercise 17 Ask students to use what they learned about similarity in Chapter 7 to explain what the label *not to scale* means.

Exercise 23 Student should recognize that 29.7° is *less than* 45° . Therefore, the height (or other leg) must be *less than* 980 ft, and answer choice D can be quickly eliminated.

GO for Help

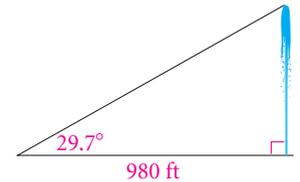
For a guide to solving Exercise 18, see p. 451.

- 18. Aerial Television** A blimp is providing aerial television views of a football game. The television camera sights the stadium at a 7° angle of depression. The blimp's altitude is 400 m. What is the line-of-sight distance from the TV camera to the stadium, to the nearest hundred meters? **3300 m**

x² Algebra The angle of elevation e from A to B and the angle of depression d from B to A are shown below. Find the measure of each angle.

- GPS** 19. $e: (7x - 5)^\circ, d: 4(x + 7)^\circ$ **72, 72** 20. $e: (3x + 1)^\circ, d: 2(x + 8)^\circ$ **46, 46**
21. $e: (x + 21)^\circ, d: 3(x + 3)^\circ$ **27, 27** 22. $e: 5(x - 2)^\circ, d: (x + 14)^\circ$ **20, 20**

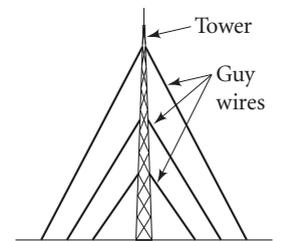
- 23. Multiple Choice** An engineer is 980 ft from the base of a fountain at Fountain Hills, Arizona. The angle of elevation to the top of the column of water is 29.7° . The surveyor's angle measuring device is at the same level as the base of the fountain. Find the height of the column of water to the nearest 10 ft. **B**
(A) 490 ft (B) 560 ft (C) 850 ft (D) 1720 ft



24a. Length of any guy wire = dist. on the ground from the tower to the guy wire div. by the cosine of the \angle formed by the guy wire and the ground.

24b. Height of attachment = dist. on the ground from the tower to the guy wire times the tangent of the \angle formed by the guy wire and the ground.

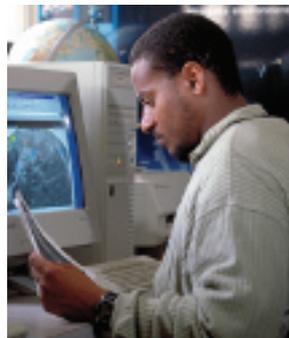
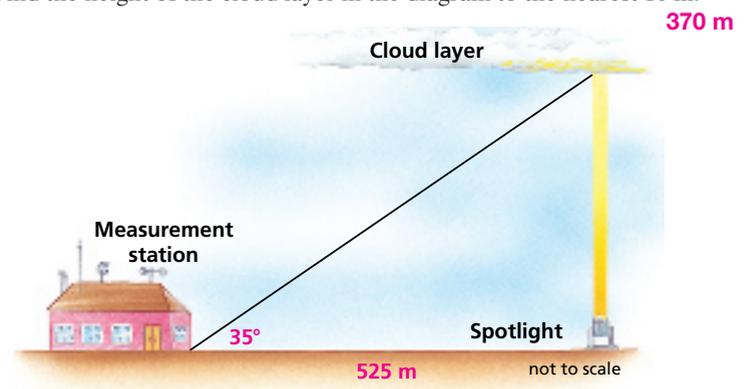
- 24. Writing** A communications tower is located on a plot of flat land. The tower is supported by several guy wires. Assume that you are able to measure distances along the ground, as well as angles formed by the guy wires and the ground. Explain how you could estimate each of the following measurements.
a. the length of any guy wire **a–b. See left.**
b. how high on the tower each wire is attached



Flying An airplane at altitude a flies distance d towards you with velocity v . You watch for time t and measure its angles of elevation, $\angle E_1$ and $\angle E_2$, at the start and end of your watch. Find the missing information.

25. $a = \blacksquare$ mi, $v = 5$ mi/min, $t = 1$ min, $m\angle E_1 = 45^\circ, m\angle E_2 = 90^\circ$ **5**
26. $a = 2$ mi, $v = \blacksquare$ mi/min, $t = 15$ s, $m\angle E_1 = 40^\circ, m\angle E_2 = 50^\circ$ **about 2.8**
27. $a = 4$ mi, $d = 3$ mi, $v = 6$ mi/min, $t = \blacksquare$ min, $m\angle E_1 = 50^\circ, m\angle E_2 = \blacksquare$ **0.5; about 84.9**

- 28. Meteorology** One method that meteorologists could use to find the height of a layer of clouds above the ground is to shine a bright spotlight directly up onto the cloud layer and measure the angle of elevation from a known distance away. Find the height of the cloud layer in the diagram to the nearest 10 m.



Real-World Connection

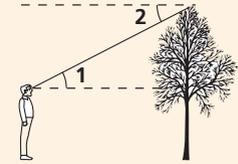
Careers Atmospheric scientists specialize by linking meteorology with another field such as agriculture.

4. Assess & Reteach

PowerPoint

Lesson Quiz

Use the diagram for Exercises 1 and 2.



- Describe how $\angle 1$ relates to the situation. **angle of elevation from man's eyes to treetop**
- Describe how $\angle 2$ relates to the situation. **angle of depression from treetop to man's eyes**

A 6-ft man stands 12 ft from the base of a tree. The angle of elevation from his eyes to the top of the tree is 76° .

- About how tall is the tree? **about 54 ft**
- If the man releases a pigeon that flies directly to the top of the tree, about how far will it fly? **about 50 ft**
- What is the angle of depression from the treetop to the man's eyes? **76°**

Alternative Assessment

Have students work in pairs to plan how to measure the height of your school building using angles of elevation and depression and trigonometric functions. Then have them carry out their plans.

Test Prep

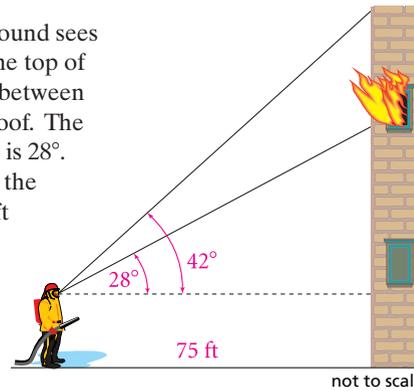


Resources

For additional test practice with a variety of test item formats:

- Standardized Test Prep, p. 465
- Test-Taking Strategies, p. 460
- Test-Taking Strategies with Transparencies

- C Challenge** 29. **Firefighting** A firefighter on the ground sees fire break through a window near the top of the building. There is voice contact between the ground and firefighters on the roof. The angle of elevation to the windowsill is 28° . The angle of elevation to the top of the building is 42° . The firefighter is 75 ft from the building and her eyes are 5 ft above the ground. What roof-to-windowsill distance can she report to the firefighters on the roof? **about 28 ft**



30. **Geography** For locations in the United States, the relationship between the latitude ℓ and the greatest angle of elevation a of the sun at noon on the first day of summer is $a = 90^\circ - \ell + 23\frac{1}{2}^\circ$. Find the latitude of your town. Then determine the greatest angle of elevation of the sun for your town on the first day of summer. **Check students' work.**



Test Prep

Multiple Choice

- A 107-ft-tall building casts a shadow of 90 ft. To the nearest whole degree, what is the angle of elevation to the sun? **C**
A. 33° B. 40° C. 50° D. 57°
- The angle of depression of a submarine from another Navy ship is 28° . The submarine is 791 ft from the ship. About how deep is the submarine? **F**
F. 371 ft G. 421 ft H. 563 ft J. 698 ft
- A kite on a 100-ft string has an angle of elevation of 18° . The hand holding the string is 4 ft from the ground. How high above the ground is the kite? **B**
A. 95 ft B. 35 ft C. 31 ft D. 22 ft

Short Response

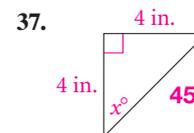
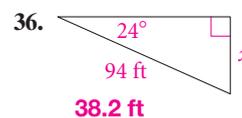
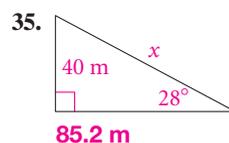
- A 6-ft-tall man is viewing the top of a tree with an angle of elevation of 83° . He is standing 12 ft from the base of the tree. **a-b. See back of book.**
 - Draw a sketch of the situation. Show a stick figure for the man. Label the angle of elevation, the height of the man, and the distance the man is standing from the tree.
 - Write and solve an equation to find the height of the tree. Round your answer to the nearest foot.

Mixed Review



Lesson 8-4

Find the value of x . Round answers to the nearest tenth.



Checkpoint Quiz

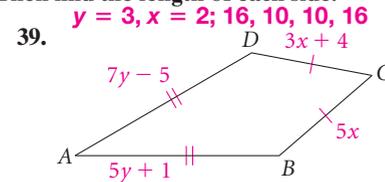
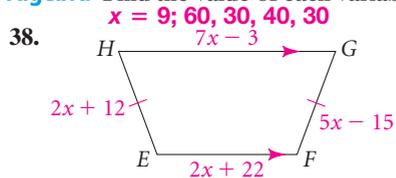
Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 8-3 through 8-5.

Resources

Grab & Go

- Checkpoint Quiz 2

Lesson 6-1 x^2 **Algebra** Find the value of each variable. Then find the length of each side.

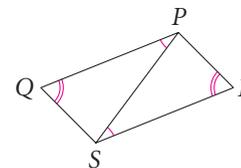


Lesson 4-4

40. **Given:** $\angle QPS \cong \angle RSP, \angle Q \cong \angle R$

Prove: $\overline{PQ} \cong \overline{SR}$

Along with Given information,
 $\overline{PS} \cong \overline{PS}$. $\triangle QPS \cong \triangle RSP$ by
AAS. $\overline{PQ} \cong \overline{SR}$ because **CPCTC**.



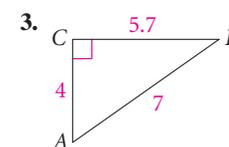
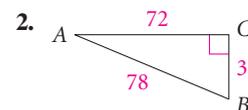
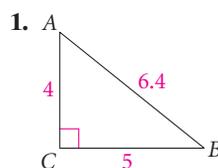
Checkpoint Quiz 1

- $\tan A = \frac{5}{4}; \sin A = \frac{25}{32};$
 $\cos A = \frac{5}{8}; \tan B = \frac{4}{5};$
 $\sin B = \frac{5}{8}; \cos B = \frac{25}{32}$
- $\tan A = \frac{5}{12}; \sin A = \frac{5}{13};$
 $\cos A = \frac{12}{13}; \tan B = \frac{12}{5};$
 $\sin B = \frac{12}{13}; \cos B = \frac{5}{13}$
- $\tan A = \frac{57}{40}; \sin A = \frac{57}{70};$
 $\cos A = \frac{4}{7}; \tan B = \frac{40}{57};$
 $\sin B = \frac{4}{7}; \cos B = \frac{57}{70}$

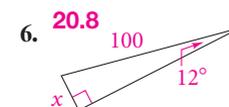
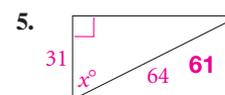
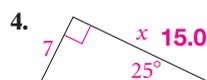
Checkpoint Quiz 2

Lessons 8-3 through 8-5

Write the tangent, sine, and cosine ratios for $\angle A$ and $\angle B$. 1-3. See margin.



x^2 **Algebra** Find the value of x . Round each segment length to the nearest tenth and each angle measure to the nearest whole number.



9. **Answers may vary.**
Sample: Identify the unknown you want to find in a right triangle. Then find two pieces of known information that will let you write a trigonometric-ratio equation you can solve for the unknown.

7. **Landmarks** The Leaning Tower of Pisa, shown at the right, reopened in 2001 after a 10-year project reduced its tilt from vertical by 0.5° . How far from the base of the tower will an object land if it is dropped the 150 ft shown in the photo? **about 13.1 ft**

8. **Navigation** A captain of a sailboat sights the top of a lighthouse at a 17° angle of elevation. A navigation chart shows the height of the lighthouse to be 120 m. How far is the sailboat from the lighthouse? **about 393 m**

9. **Writing** How do you decide which **See left.** trigonometric ratio to use to solve a problem?

10. **Hang Gliding** Students in a hang gliding class stand on the top of a cliff 70 m high. They watch a hang glider land on the beach below. The angle of depression to the hang glider is 72° . How far is the hang glider from the base of the cliff? **about 22.7 m**

